Numbers Divisible by 3

Any number is divisible by 3 if the sum of the digits is divisible by 3. For example 123 is 1+2+3 = 9 so it is divisible by 3.

Proof:

Two Digit Proof:

$n = 10a + b$ Let n be the number, a and b the digits

$a + b = c$ The sum of the digits is c

$b = c - a$ Re-arrange the sum of the digits

$n = 10a + c - a$ Substitute into the first equation

$n = 9a + c$ Simplify

Given $a + b \ \ \%3 = 0$, or the sum of the digits is divisible by 3, c is divisible by 3

Both 9a and c are divisible by 3, allowing a 3 to be factored out, guaranteeing $n\%3 = 0$

Three Digit Proof:

$n = 100a + 10b + c$ Let n be the number, a, b, and c the digits

$a + b + c = d$ The sum of the digits is d

$c = d - b - a$ Re-arrange the sum of the digits

$n = 100a + 10b + d - b - a$ Substitute into the first equation

$n = 99a + 9b + d$ Simplify

Given $a + b + c \ \ \%3 = 0$, or the sum of the digits is divisible by 3, d is divisible by 3

99a, 9b, and d are divisible by 3, allowing a 3 to be factored out, guaranteeing $n\%3 = 0$

X Digit Proof:

$digit_0$ is the least significant digit, $digit_x$ is the most significant digit

$n = \sum_{i=0}^{x} digit_i 10^i$ Let n be the number and $digit_i$ the digits

$s = \sum_{i=0}^{x} digit_i$ The sum of the digits is s

$digit_0 = s - \sum_{i=1}^{x} digit_i$ Re-arrange the sum of the digits

$n = \sum_{i=1}^{x} \ digit_i 10^i \ \ + digit_0 10^0$ Re-write the first equation

$n = \sum_{i=1}^{x} \ digit_i 10^i \ \ + digit_0$ Simplify

$n = \sum_{i=1}^{x} \ digit_i 10^i \ \ + s - \sum_{i=1}^{x} digit_i$ Substitute the re-arranged sum of the digits

$n = \sum_{i=1}^{x} \ digit_i 10^i - digit_i \ \ + s$ Combine the summations

$$n = \sum_{i=1}^{x} digit_i \;\; 10^i - 1 \;\; + s$$ Factor

Need to prove $\forall i \;\; 10^i - 1 \;\; \% 3 = 0$

$$10^i - 1 \;\; = 10^{i-1+1} - 1$$ Add and subtract 1 from the exponent

$$10^{i-1+1} - 1 = \frac{10^{i-1+1} - 1}{9} \;\; 9$$ Multiply and divide by 9

$$\frac{10^{i-1+1} - 1}{9} \;\; 9 \;\; = \;\; 9 \;\; \frac{10^{i-1+1} - 1}{10 - 1}$$ Re-write the bottom 9 as (10 – 1)

$$9 \;\; \frac{10^{i-1+1} - 1}{10 - 1} = 9 \sum_{z=0}^{i-1} 10^z$$ Re-write equation as a summation

$$n = \sum_{i=1}^{x} \left(digit_i 9 \sum_{z=0}^{i-1} 10^z \right) + s$$ Replace original equation with summation

$$n = 9 \sum_{i=1}^{x} \left(digit_i \sum_{z=0}^{i-1} 10^z \right) + s$$ Pull the 9 out of the summation

9 and s are divisible by 3, allowing a 3 to be factored out, guaranteeing $n \% 3 = 0$